Characterization of algebraic varieties by their groups of symmetries.

Abstract:

An old question steaming from Klein's Erlangen Program can be phrased in modern terms as: Is a given geometric object uniquely determined by its group of symmetries? The first part of this talks consists of an introduction to the problem for a general audience and with some selected examples from outside algebraic geometry.

In the second part of the talk we come to the setting of algebraic geometry, where we show that, in general, the answer to the above question is negative. After restricting the class of varieties we will show an instance where the answer is affirmative. Indeed, we show that complex affine toric surfaces are determined by the abstract group structure of their regular automorphism groups in the category of complex normal affine surfaces using properties of the Cremona group.