

Mixed weak-type inequalities for maximal functions, Calderón-Zygmund operators and their commutators

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In [2], E. Sawyer proved a mixed weak-type inequality for the one-dimensional Hardy-Littlewood maximal operator M , inspired by the main ideas of the principal intervals developed in [1]. Specifically, if u and v are A_1 weights, the result in [2] established that the operator $S_v(f) = M(fv)/v$ is bounded from $L^1(uv)$ into $L^{1,\infty}(uv)$. Sawyer's motivation for considering this type of estimate was that it yields a new proof of the classical boundedness of M on $L^p(w)$ for $w \in A_p$. Since then, several authors have focused on these types of inequalities, providing alternative results under different conditions on the weights and considering various operators from harmonic analysis. In this lecture, we give an overview of these developments, including some new results regarding mixed weak-type inequalities for maximal functions, Calderón-Zygmund operators and their commutators with BMO symbols. We also discuss weighted mixed estimates recently obtained for pairs of weights.

Referencias

- [1] Muckenhoupt, B., Wheeden, R., Some weighted weak-type inequalities for the Hardy-Littlewood maximal function and the Hilbert transform, *Indiana Univ. Math. J.* 26 (1977), no. 5, 801-816.
- [2] Sawyer, E.: A weighted weak type inequality for the maximal function. *Proc. Amer. Math. Soc.* 93 (1985), no. 4, 610-614.