On the inverse of nonsingular unicyclic graphs.

Expositor: Daniel A. Jaume (Universidad Nacional de San Luis, djaume@unsl.edu.ar)
Autor/es: Daniel A. Jaume (Universidad Nacional de San Luis, djaume@unsl.edu.ar); Gonzalo Molina (Universidad Nacional de San Luis, gonzalo.molina.tag@gmail.com); Cristian Panelo (Universidad Nacional de San Luis, cristian.panelo.tag@gmail.com); Rodrigo Sota (Universidad Nacional de San Luis, rodrigo.sota.tag@gmail.com)

An unicyclic graph is a connected graph containing exactly one cycle. The induced cycled in U is denoted by C. An unicyclic graph U is said that is nonsingular if its adjacency matrix $A(U)$ is nonsingular. Unicyclic graphs are of three types: T1, T2 and T3. In this work we give formulas for the inverse of nonsingular unicyclic in terms of their matching structure.

Given an unicyclic graph U, a walk W in U and a matching M of U, the walk W is called an alternating walk with respect to M if it has edges that are alternately unmatched and matched in M. An alternating walk W with respect to M is said a coaugmenting walk in U with respect to M if W starts and ends at matched edges. The set of all coaugmenting walks from i to j (two different vertices of U) with respect to any maximun matching M of U is denoted by $CoW(U,i,j)$.

Let U be a nonsingular unicyclic graph of Type 1 of order n. Then

$$A^{-1}(U) = Inv_1(U) := \sum_{W \in CoW(U,i,j)} (-1)^{\left\lfloor \frac{|W|}{2} \right\rfloor},$$

where $|W|$ is the length of the walk W.

For each vertex $v \in V(C)$, $T(v)$ denotes the pendant tree at v. We define $PSupp(U) := \bigcup_{u \in V(C)} Supp(T(u))$. A matching M of U is called non-sun matching if $e \in M$ and $|e \cap E(C)| > 1$, then $|e \cap E(C)| = 2$. Let $i,j \in V(U)$, the set of all the coaugmenting walks from i to j with respect to any non-sun maximum matching is denoted by $CoW_{ns}(U,i,j)$. These walks are called non-sun coaugmenting walks.

For every nonsingular unicyclic U of type 2 $A(U)^{-1} = Inv_1(U) + Inv_2(U)$, where $Inv_1(U)$ and $Inv_2(U)$ are two matrices of order n given by

$$Inv_1(U)_{i,j} := \begin{cases} \frac{1}{2}(-1)^{\left\lfloor \frac{d_w(i,j)}{2} \right\rfloor}, & \text{if } i,j \in PSupp(G) \text{ and } CoW_{ns}(U,i,j) \neq \emptyset, \\ \\ \frac{1}{2}(-1)^{\left\lfloor \frac{|C|}{2} \right\rfloor + \left\lfloor \frac{d(i,j)}{2} \right\rfloor}, & \text{if } i,j \in PSupp(G) \text{ and } CoW_{ns}(U,i,j) = \emptyset, \\ \\ 0, & \text{otherwise,} \end{cases}$$

where $d_w(i,j)$ is the length of a shorter non-sun coaugmenting walk from i to j, and $d(i,j)$ is the usual distance. And

$$Inv_2(U)_{i,j} := \begin{cases} \sum\limits_{W \in CoW(U,i,j)} (-1)^{\left\lfloor \frac{|W|}{2} \right\rfloor}, & \text{if } i,j \in V(U-C) \text{ and } CoW(U-C,i,j) \neq \emptyset, \\ \\ 0, & \text{otherwise.} \end{cases}$$

For every nonsingular unicyclic U of type 3 $A(U)^{-1} = Inv_2(U) + Inv_3(U)$, where $Inv_3(U)$ is the matrix of order n given by

$$Inv_3(U)_{i,j} := \begin{cases} \frac{1}{2}(-1)^{\left\lfloor \frac{d_w(i,j)}{2} \right\rfloor}, & \text{if } i,j \in PSupp(G) \text{ and } CoW(U,i,j) \neq \emptyset, \\ \\ 0, & \text{otherwise.} \end{cases}$$