Independence and Matching Numbers of Unicyclic Graphs from Null Space

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Let G be a graph with n vertices. The support of the null space of $A(G)$ is denoted by $Supp(G)$. Let T be a tree. The S-forest of T, denoted by $\mathcal{F}_S(T)$, is defined as the subgraph induced by the closed neighborhood of $Supp(T)$. The N-forest of T, denoted by $\mathcal{F}_N(T)$, is defined as $\mathcal{F}_N(T) := T - \mathcal{F}_S(T)$. The core of G, denoted by $Core(G)$, is the set of all the neighbors of the supported vertices of G.

A unicyclic graph G is a connected graph containing exactly one cycle. The induced cycle in G is denoted by C. A pendant tree of G at $v \in V(C)$, denoted $G\{v\}$, is the induced connected subgraph of G with maximum possible number of vertices, which contains the vertex v and no other vertex of C.

A unicyclic graph G is of Type I if and only if there exists at least one pendant tree $G\{v\}$ such that $v \notin Supp(G\{v\})$. A unicyclic graph G is of Type II if and only if every pendant tree $G\{v\}$ is such that $v \in Supp(G\{v\})$.

Let G be a unicyclic graph and C its cycle. Let $G - C = \bigcup_{i=1}^{k} T_i$, where T_i is a connected component of $G - C$.

We show that, if G is a unicyclic graph of Type I and $G\{v\}$ its pendant tree such that $v \notin Supp(G\{v\})$, then

$$\alpha(G) = |Supp(G\{v\})| + |Supp(G - G\{v\})| + \frac{|V(\mathcal{F}_N(G\{v\}))| + |V(\mathcal{F}_N(G - G\{v\}))|}{2},$$

$$\nu(G) = |Core(G\{v\})| + |Core(G - G\{v\})| + \frac{|V(\mathcal{F}_N(G\{v\}))| + |V(\mathcal{F}_N(G - G\{v\}))|}{2},$$

and if G is a unicyclic graph of Type II, then

$$\alpha(G) = \left\lfloor \frac{|V(C)|}{2} \right\rfloor + \sum_{i=1}^{k} |Supp(T_i)| + \frac{|V(\mathcal{F}_N(T_i))|}{2},$$

$$\nu(G) = \left\lfloor \frac{|V(C)|}{2} \right\rfloor + \sum_{i=1}^{k} |Core(T_i)| + \frac{|V(\mathcal{F}_N(T_i))|}{2}.$$