The obstacle problem for a class of degenerate fully nonlinear operators

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We study the obstacle problem for fully nonlinear elliptic operators with an anisotropic degeneracy on the gradient:

$$\begin{cases} \text{mín}\left\{f - |Du|^\gamma F(D^2u), u - \phi\right\} & = & 0 & \text{in } \Omega \\ u & = & g & \text{on } \partial\Omega. \end{cases}$$

We obtain existence of solutions and prove sharp regularity estimates along the free boundary points, namely $\partial\{u > \phi\} \cap \Omega$. In particular, for the homogeneous case ($f \equiv 0$) we get that solutions are $C^{1,1}$ at free boundary points, in the sense that they detach from the obstacle in a quadratic fashion, thus beating the optimal regularity allowed for such degenerate operators. We also present further features of the solutions and partial results regarding the free boundary.

These are the first results for obstacle problems driven by degenerate type operators in non-divergence form and they are a novelty even for the simpler scenario given by an operator of the form $\mathcal{G}[u] = |Du|^\gamma \Delta u$.