On the uniqueness of bound state solutions of a semilinear equation with weights

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We consider radial solutions of a general elliptic equation involving a weighted Laplace operator. We establish the uniqueness of the radial bound state solutions to

$$\operatorname{div}\big(\mathsf{A}\,\nabla v\big) + \mathsf{B}\,f(v) = 0\,, \qquad \lim_{|x|\to+\infty} v(x) = 0, \quad x \in \mathbb{R}^n, \tag{P}$$

$n > 2$, where A and B are two positive, radial, smooth functions defined on $\mathbb{R}^n \setminus \{0\}$. We assume that the nonlinearity $f \in C(-c, c)$, $0 < c \leq \infty$ is an odd function satisfying some convexity and growth conditions, and has a zero at $b > 0$, is non positive and not identically 0 in $(0, b)$, positive in (b, c), and is differentiable in $(0, c)$.