On the mean field equation with variable intensities on pierced domains.

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We consider the mean field equation of the equilibrium turbulence with variable intensities and Dirichlet boundary condition on a pierced domain $\Omega_\varepsilon := \Omega \setminus \cup_{i=1}^m \overline{B(\xi_i, \varepsilon_i)}$:

$$\begin{cases} -\Delta u = \lambda_1 \dfrac{V_1(x)e^u}{\int_{\Omega_\varepsilon} V_1(z)e^u dz} - \lambda_2 \tau \dfrac{V_2(x)e^{-\tau u}}{\int_{\Omega_\varepsilon} V_2(z)e^{-\tau u} dz} & \text{in } \Omega_\varepsilon, \\ u = 0 & \text{on } \partial\Omega_\varepsilon, \end{cases} \tag{1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^2$, $B(\xi_i, \varepsilon_i)$ is a ball centered at $\xi_i \in \Omega$ with radius $\varepsilon_i = \varepsilon_i(\varepsilon) > 0$, $i = 1, \ldots, m$, depending on some $\varepsilon > 0$ small enough, V_1 and V_2 are positive smooth bounded functions in $\bar{\Omega}$ and $\tau > 0$. Given m different points ξ_i, $i = 1, \ldots, m$, $\lambda_1 > 8\pi m_1$ and $\lambda_2 \tau^2 > 8\pi m_2$ with $m = m_1 + m_2$ and $m_i \in \mathbb{N} \cup \{0\}$, we have found suitable radii ε_i, $i = 1, \ldots, m$ such that for $\varepsilon > 0$ small enough problem (1) has a solution u_ε in Ω_ε blowing up positively around each $\xi_1, \ldots, \xi_{m_1}$ and negatively around $\xi_{m_1+1}, \ldots, \xi_m$ respectively, as $\varepsilon \to 0$. We have used a family of solutions of the singular Liouville equation

$$\Delta u + |x - \xi|^{\alpha-2} e^u = 0 \quad \text{in } \mathbb{R}^2, \quad \text{satisfying} \quad \int_{\mathbb{R}^2} |x - \xi|^{\alpha-2} e^u < +\infty$$

to construct an approximation of the solution depending on some parameters, suitable projected and scaled in order to make the error small enough in a suitable norm. Hence, we have found an actual solution as a small additive perturbation of this initial approximation by using a perturbative approach and a fixed point argument.