Decay of small odd solutions of the long range Schrödinger and Hartree equations in one dimension

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We consider the long time asymptotics of (not necessarily small) odd solutions to the one-dimensional nonlinear Schrödinger equation

$$iu_t + u_{xx} = g(u), \quad (t,x) \in \mathbb{R} \times \mathbb{R}. \tag{1}$$

with semi-linear nonlinearities

$$g(u) = \mu V(x)u + |u|^{p-1}u, \quad 1 < p < 5, \tag{2}$$

where the potential V is a Schwartz even function, and nonlocal Hartree nonlinearity

$$g(u) = \left(\frac{1}{|x|^a} * |u|^2\right)u, \quad 0 < a < 1. \tag{3}$$

We assume data in the energy space only and we prove decay to zero in compact regions of space as time tends to infinity. We give three different results were decay holds: NLS without potential, NLS with potential and Hartree (defocusing case). The proof is based in the use of suitable virial identities and covers all range of scattering sub, critical and supercritical (long range) nonlinearities.