Analysis of a momentum conservative mixed-FEM for the stationary Navier-Stokes problem

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In this work we propose and analyze a new momentum conservative mixed finite element method for the Navier–Stokes problem posed in non-standard Banach spaces. Our approach is based on the introduction of a pseudostress tensor relating the velocity gradient with the convective term, leading to a mixed formulation where the aforementioned pseudostress tensor and the velocity are the main unknowns of the system. Then the associated Galerkin scheme can be defined by employing Raviart–Thomas elements of degree k for the pseudostress tensor and discontinuous piece–wise polynomial elements of degree k for the velocity. With this choice of spaces, the equilibrium equation is exactly satisfied if the external force belongs to the velocity discrete space, thus the method conserves momentum, which constitutes one of the main feature of our approach. For both, the continuous and discrete problems, the Banach–Nečas–Babuška and Banach's fixed point theorems are employed to prove unique solvability. We also provide the convergence analysis and particularly prove that the error decay with optimal rate of convergence. Further variables of interest, such as the fluid pressure, the fluid vorticity and the fluid velocity gradient, can be easily approximated as a simple postprocess of the finite element solutions with the same rate of convergence. Finally, several numerical results illustrating the performance of the method are provided.