Analysis of a mixed-FEM for stationary incompressible magneto-hydrodynamics

Expositor: Ricardo Oyarzúa (Universidad del Bio-Bio, royarzua@ubiobio.cl)
Autor/es: Ricardo Oyarzúa (Universidad del Bio-Bio, royarzua@ubiobio.cl); Jessika Camaño (Universidad Catolica de a Santisima Concepcion, jecamano@ucsc.cl); Carlos Garcia (Pontificia Universidad Catolica de Chile, cgarciv@mat.uc.cl)

In this paper we propose and analyze a new mixed finite element method for the stationary incompressible magneto-hydrodynamics. The method is based on the introduction of a pseudostress tensor relating the velocity gradient with the convective term, leading to a mixed formulation where the aforementioned pseudostress tensor and the velocity are the main hydrodynamic unknowns, while the magnetic field and a Lagrange multiplier are the magnetic unknowns. Then the associated Galerkin scheme can be defined by employing Raviart-Thomas elements of degree k for the pseudostress tensor, discontinuous piecewise polynomial elements of degree k for the velocity, Nédélec elements of degree k for the magnetic field and Lagrange elements of degree k for the respective Lagrange multiplier. The analysis of the continuous and discrete problems are carried out by means of the Lax–Milgram lemma, the Banach–Nečas–Babuška theorem and the Banach fixed-point theorem, under a sufficiently small data assumption. We also develop an a priori error analysis and show that the proposed finite element approximation leads to optimal order of convergence.