Formation of $\mathbb{Z}^2$-crystals under one-well interaction potentials

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We study N-point configurations of points which minimize an energy of the form $\mathcal{E}[V](X) := \sum_{1\leq i<j\leq N} V(|X(i) - X(j)|)$, in which V is a pairwise interaction potential with one well, and $X : \{1, \dots, N\} \to \mathbb{R}^2$ is a configuration of N particles.

The geometric structure of minimizing configurations was first described by Heitmann-Radin 1981 and Theil 2006. They give conditions on V under which $\mathcal{E}[V]$-minimizers tend to a triangular lattice as $N \to \infty$. Theil 2006 also presents numerical evidence that the square lattice $\mathbb{Z}^2$ should appear for other one-well V's. I will present recent work with L. Betermin and L. De Luca, in which we give for the first time a rigorous proof of such square-lattice crystallization. Some seemingly robust new principles and ideas are required in the proof.

Since our new potentials are obtained via a simple modification of the classical ones (basically, we enlarge the width of the "well.ºf V), this also shows in particular that a phase trasition occurs, from triangular to square lattice structures. If time allows, I will present several new directions and challenges opened by our method of proof.