CONNECTIVITY OF THE REAL AND THE BRANCH LOCUS IN MODULI SPACE $\mathcal{M}_{0,[n+1]}$

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Let $\mathcal{M}_{0,[n+1]}$ be the moduli space of isomorphisms classes of $(n+1)$-marked spheres, where $n \geq 3$. It is know that $\mathcal{M}_{0,[n+1]}$ has a complex orbifold structure of dimension $n-2$. Moreover, the space $\mathcal{M}_{0,[n+1]}$ admits a natural real structure $\hat{J}$, this being induced by the complex conjugation on the Riemann sphere. The fixed points of $\hat{J}$ are called the real points and these points corresponds to the classes of isomorphisms of marked spheres admitting an anticonformal automorphism. Inside this locus is the real locus $\mathcal{M}^{\mathbb{R}}_{0,[n+1]}$, consisting of those classes of marked spheres admitting an anticonformal involution. Let us denote by $\mathcal{B}_{0,[n+1]}$ the branch locus of $\mathcal{M}_{0,[n+1]}$ (the isomorphism classes of those $(n+1)$-marked spheres with non-trivial group of conformal automorphisms). It is known that $\mathcal{B}_{0,[4]} = \mathcal{M}_{0,[4]}$ (as any collection of four points in the Riemann sphere is invariant by a subgroup of Möbius transformations isomorphic to $\mathbb{Z}_2^2$) and that $\mathcal{B}_{0,[n+1]} \neq \mathcal{M}_{0,[n+1]}$ for $n \geq 4$.

The main aim of this talk is to observe the following:

1. $\mathcal{B}_{0,[n+1]}$ is connected if either (i) $n \geq 4$ is even or (ii) if $n \geq 6$ is divisible by 3. It has exactly two connected components otherwise.

2. $\mathcal{M}^{\mathbb{R}}_{0,[n+1]}$ is connected for $n \geq 5$ odd. It is disconnected for $n = 2r$ with $r \geq 5$ odd.

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