

DOMINATING K_t MODELS IN KNESER GRAPHS

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Hadwiger's conjecture (1943) states that every graph G has a $K_{\chi(G)}$ minor, where $\chi(G)$ denotes the chromatic number of G . This is a generalization of the Four Color Theorem. Over the years, several strengthenings and weakenings of the conjecture have been studied. The problem of dominating K_t models is one such strengthening.

A *dominating K_t model* is a sequence $(T_1, \dots, T_t)$ such that each T_i is a connected subgraph and, for every $1 \leq i \leq j \leq t$, every vertex of T_j has a neighbor in T_i . Dominating K_t models were introduced in [1] as a stronger version of Hadwiger's conjecture.

The Kneser graph $KG(2k+r, k)$ has as its vertices the k -subsets of a set of size $2k+r$, with two vertices adjacent if and only if the corresponding subsets are disjoint. Kneser graphs are interesting from a topological point of view and have played an important role in the study of variations of Hadwiger's conjecture. In particular, they provide a natural setting in which to investigate stronger notions of complete minors. Recall that $\chi(KG(2k+r, k)) = r+2$. *Recent results on odd subdivisions, which provide a stronger structure than a complete minor for $k \geq 13$ and $r \geq 6$, with $2r \mid (k-1)$, $KG(2k+r, k)$ contains $K_{\lceil (k+r)/8 \rceil}$ as a totally odd subdivision.*

In this talk, we study dominating K_t models in Kneser graphs. We present two such models. The first has size $\lceil t = (r-2\ell-2) \cdot \min \binom{k+\ell-1}{k-2}, \binom{k+3\ell-3}{k-2} - \binom{k+\ell-2}{k-2} \rceil$ for every positive integer ℓ such that $r-2\ell-3 \geq 0$. The second has size $\lceil k + \lceil r/2 \rceil \binom{k+1}{k+1} \rceil$. *These two constructions are complementary: depending on the relative size of r and k , one of them can provide a better bound.*

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Referencias

[1] Illingworth, F. and Wood, D.R. (2025), Dominating K_t -Models. Journal of Graph Theory, 110: 448-456.
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