Some categorical constructions combining propositional logics

Ciro Russo
Universidade Federal da Bahia - Departmento de Matemática, Brasil
ciro.russo@ufba.br

In [2], we defined the category $\mathcal{QM}$, whose objects are pairs (Q, M) where Q is a quantale and M a left Q-module. The morphisms of $\mathcal{QM}$ are pairs $(h, f) : (Q, M) \to (R, N)$, of a quantale morphism h and a Q-module morphism f, with N becoming a Q-module by restricting the scalars along h. Then we defined a non-full subcategory of $\mathcal{QM}$, denoted by $\mathcal{DS}_0$, which is equivalent to the category whose objects are propositional deductive systems and whose morphisms are interpretations (up to translations) among them. It turns out that various constructions aimed at combining different logics, presented in [1], are indeed classical categorical constructions in $\mathcal{DS}_0$, such as coproduct, pushout, and amalgamation.

In this talk, we shall recall the main results of both [1] and [2], and we will present drafts of other constructions that are currently being studied (jointly with Júlia Cardoso Penha), such as products and colimits.

Referencias

[1] Russo, C.; Coproduct and Amalgamation of Deductive Systems by Means of Ordered Algebras. Logica Universalis, 16 (2022), pp. 355-380

[2] Russo, C.; The Category of Propositional Deductive Systems. The Review of Symbolic Logic. Published online 2026:1-16. doi:10.1017/S1755020326101245