On the limits of paraconsistency in LFIs with replacement

Federico Mallea
Universidade Estadual de Campinas (UNICAMP) - Centro de Lógica, Epistemologia e História da Ciência (CLE), Brasil
hfmallea@gmail.com

RmbC is a paraconsistent logic in the family of *logics of formal inconsistency* (LFIs). It is equipped with a negation $\neg$ that is non-explosive and with a consistency operator $\circ$ that allows explosion to be recovered in a controlled way. The logic is obtained from mbC, the basic LFI, by adding the replacement property by means of two global inference rules. This property makes it possible to algebraize RmbC via the variety of Boolean algebras with LFI operators, or BALFIs. RmbC, together with a family of axiomatic extensions of it, was introduced by Carnielli, Coniglio, and Fuenmayor in [1], and constitutes a natural framework for studying the algebraic behavior of paraconsistency.

In this talk we analyze the landscape of the main axiomatic extensions of RmbC and their associated algebraic semantics. The aim is to describe to what extent paraconsistency can be preserved by adding to RmbC the usual axioms of consistency and propagation [2, 3], and to identify precisely the threshold beyond which paraconsistency is lost, i.e., classical logic is attained.

We classify the combinations of usual axioms that, added to RmbC, produce classical logic (equivalently, those whose corresponding subvariety of BALFIs satisfies the equation $x \wedge \neg x = 0$), and identify two completely independent collapse mechanisms. The first is equational in nature: certain combinations of consistency-propagation axioms, such as $(\circ\alpha \wedge \circ\beta) \to \circ(\alpha \# \beta)$ for $\# \in \{\wedge, \vee, \to\}$, together with axioms that bring paraconsistent negation closer to classical negation, give rise, in the algebraic setting, to subvarieties in which $\neg$ coincides with the Boolean complement and $\circ x = 1$ for every x. The second mechanism is combinatorial and non-equational: in every finite Boolean algebra, a negation satisfying simultaneously $\neg\neg x = x$ and $x \vee \neg x = 1$ must necessarily coincide with the Boolean complement. Since the axioms (ce) $\alpha \to \neg\neg\alpha$ and (cf) $\neg\neg\alpha \to \alpha$ are algebraically equivalent to the equation $\neg\neg x = x$, it follows that no finite paraconsistent BALFI satisfies simultaneously the equations associated with both axioms, although there do exist infinite paraconsistent models that do satisfy them.

As an application, we consider Cila, the presentation of da Costa's logic C_1 in the signature of LFIs. It is known that its self-extensional version is not paraconsistent [1]. We introduce C_1^w, a natural weaker version of Cila, obtained by removing exactly the axioms responsible for the collapse and adding suitable axioms to preserve the full propagation of consistency. Thus, we show that the logic C_1^w does preserve paraconsistency when the replacement property is added.

These results precisely delimit how much of the standard hierarchy of consistency principles for LFIs remains after the passage to a genuinely self-extensional paraconsistent negation.

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Referencias

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