

METRIC STRUCTURES AND THE KEISLER-SHELAH THEOREM

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In the 2000s, a continuous model theory based on metric structures emerged. This framework proves to be fruitful for geometric and analytic theories, such as Banach lattices and C^* -algebras, and, more broadly, within the realm of functional analysis. Unlike in classical model theory, formulae are interpreted in complete and bounded metric spaces, incorporating the metric as a binary relation that replaces equality in classical logic. From this perspective, classical model theory can be viewed as a restriction to structures with the discrete metric. In this presentation, we proffer a comprehensive introduction to the theory, culminating in some fundamental results. The main objective is to present the Keisler-Shelah theorem, which guarantees that elementarily equivalent structures have isomorphic ultrapowers. In the absence of a self-contained proof of this pivotal theorem in the literature, we provide such a demonstration within this framework. To achieve this, we follow Shelah's construction for the classical case, making the requisite adaptations and appropriating machinery from transfinite set theory to inductively construct an ultrafilter that witnesses the existence of isomorphic ultrapowers. This communication is based on the thesis [1] presented by the author in his Master's studies and was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

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Referencias

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