

A MODAL EXPANSION OF KLEENE ALGEBRAS VIA TWIST STRUCTURES

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Jalali's representation of Kleene algebras by means of triples $\langle D, R, F \rangle$, where D is a bounded distributive lattice, R is an independence relation dual to contact, and F is a Boolean filter [1], yields a categorical equivalence between Kleene algebras and a category of triples. In this talk we study how this representation can be enriched by incorporating modal operators on the distributive lattice component. This leads to the notion of *modal triples*, extending the framework of Kleene triples in a way analogous to the modal representation recently introduced for Nelson algebras [2]. We show that modal triples yield a class of modal Kleene algebras and establish a categorical equivalence between these two structures.

The modal triple framework also provides a natural setting for studying implication. Motivated by the fact that, in Heyting algebras, every implication determines a pair of modal operators, we show that an implication on the lattice component gives rise to a parametric family of modal operators. As a consequence, the modal triple framework provides a representation of Kleene algebras with implication, yielding a categorical equivalence between lattices with implication and Kleene algebras equipped with implication.

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Referencias

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- [2] Menchón, P. and Rodriguez, R. O., Twist-structures isomorphic to modal Nelson lattices, *arXiv:2504.08109*, 2025.