The tentacles landscape

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Basins of attraction in multistable high dimensional dynamical systems are expected to have universal features but very little has been proved rigorously. We'll consider phase oscillators coupled according to a cycle graph, $\dot{\theta}_i = f(\theta_{i+1} - \theta_i) + f(\theta_{i-1} - \theta_i)$, with coupling f that is C^1, odd, 2π-periodic, and strictly increasing on $(-\pi, \pi)$. We'll prove the full "octopus" picture of the basins of attraction observed numerically by Zhang and Strogatz [Phys. Rev. Lett. 127 (2021) 194101] and, beyond this model, across a wide family of high-dimensional multistable systems.

In our case, we have a family of stable equilibria that can be indexed by their winding number $q \in \mathbb{Z} \cap (-n/2, n/2)$. Basin volumes obey a Gaussian law $\mu(\mathcal{K}_q) = \sqrt{6/(\pi n)}\, e^{-6q^2/n}(1 + o(1))$ in the winding number. The distance from a uniform sample to its attractor, when divided by $\sqrt{n}$, concentrates at $\sqrt{\pi^2/3} \approx 1{,}814$. Along almost every straight line through any twisted state, the ray enters every other basin infinitely many times, with frequencies given by the basin volumes. The inscribed ball at a twisted state has radius $(\pi/\sqrt{2})(1 - 2|q|/n)$ for every n, while as $n \to \infty$, a typical ray travels distance $(\pi/2)\sqrt{n/\log n}$ before first leaving the basin: the head of the octopus is sharply anisotropic.

Referencias

[1] Yuanzhao Zhang and Steven H Strogatz, *Basins with tentacles*, Phys. Rev. Lett. **127** (2021), no. 19, 194101.