A Generalized Nonlocal Quasilinear Problem in Orlicz Spaces

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A fractional counterpart of the (p,q)-Laplacian is given by $(-\Delta_p)^s u + (-\Delta_q)^s u$, where

$$(-\Delta_p)^s u(x) := C(n,s)\,\text{p.v.} \int_{\mathbb{R}^n} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{n+sp}}\,dy.$$

Several generalizations of this operator have been proposed in the setting of Fractional Orlicz spaces; however, these constructions breaks down when the two terms are allowed to have different fractional orders. To cover this and more general situations, in this talk, for $s_1,\dots,s_k \in (0,1)$, we introduce a vector-valued fractional gradient $D^{\mathbf{s}}u = (D^{s_1}u,\dots,D^{s_k}u)$ where $\mathbf{s} = (s_1,\dots,s_k)$ and study energies for a generalized N-function

$$M : \mathbb{R}^k \to \mathbb{R}_+ \qquad \iint_{\mathbb{R}^{2n}} M(D^{\mathbf{s}}u)\,d\nu_n \quad \text{where} \quad D^s u(x,y) = \frac{u(x)-u(y)}{|x-y|^s} \quad \text{and} \quad d\nu_n = \frac{dxdy}{|x-y|^n}.$$

We then develop the corresponding fractional Orlicz–Sobolev spaces and establish classical results in this context, such as a Poincaré inequality and a Rellich–Kondrashov-type compactness theorem. Finally, we present some existence results for equations involving this new operator.

This is joint work with Julián Fernández Bonder (IC-UBA) and Laura Cuneo (IC-UBA).