Multi-Bump Solutions for Anisotropic Elliptic Problems

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In this talk we study the existence, multiplicity, and concentration of positive solutions for a class of quasilinear elliptic problems governed by the anisotropic p-Laplacian

$$-\Delta_{\vec{p}} u + (\lambda V(x) + Z(x))u^{p_0 - 1} = u^{q-1}, \quad u > 0 \text{ in } \mathbb{R}^N,$$

where, for a vector of exponents $\vec{p} = (p_1, \dots, p_N)$, the anisotropic operator is defined as

$$-\Delta_{\vec{p}} u := -\sum_{i=1}^{N} \partial_{x_i} \left(|\partial_{x_i} u|^{p_i - 2} \partial_{x_i} u \right),$$

$\lambda > 0$ is a parameter, and the potentials V, Z are continuous, nonnegative, satisfy $\lambda V(x) + Z(x) \geq M_0 > 0$ for all $x \in \mathbb{R}^N$ and all $\lambda \geq 1$, and are such that the set where V vanishes has k disjoint connected components (the potential wells).

We extend to the anisotropic setting the variational techniques developed in [1] for the isotropic case (when all exponents $p_1, \dots, p_N$ coincide). A truncated energy functional is constructed on the weighted anisotropic Sobolev space E_λ, and it is shown that this functional satisfies the Palais-Smale condition for every $\lambda \geq 1$. Using multidimensional minimax theory, we prove that for λ sufficiently large the problem admits at least $2^k - 1$ positive solutions, where k is the number of wells. Moreover, we establish that the resulting families of solutions concentrate, as $\lambda \to \infty$, on the chosen components of the wells, converging to least energy solutions of the corresponding limit problems.

Trabajo en conjunto con Pablo Ochoa (Universidad Nacional de Cuyo-CONICET, Argentina) y Analía Silva (Universidad Nacional de San Luis-CONICET, IMASL, Argentina).

Referencias

[1] C.O. Alves, Existence of multi-bump solutions for a class of quasilinear problems, Advanced Nonlinear Studies 6 (2006), 491-509.