Stabilizing Control Parameters Values for the HMN Problem

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The Huijberts–Michiels–Nijmeijer (HMN) problem addresses the stabilization of a single-input single-output (SISO) linear system on its equilibrium point.

In this work, we consider the controllable two-dimensional case in its canonical form, given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_1 x_1 - a_2 x_2 - u \end{cases} \tag{1}$$

with output:

$$y = c_1 x_1 + c_2 x_2. \tag{2}$$

System (1) with $u \equiv 0$ has $x^* = (0, 0)$ as its unique equilibrium point. In order to stabilize this system at x^*, Pyragas control is proposed:

$$u(t) = k(y(t - \tau) - y(t)) \tag{3}$$

where k is the feedback gain and τ is the time delay.

In [1] and [2], the description of the stabilization regions of the controlled system (1)–(2)–(3) is addressed—that is, to determine for which system parameter values (a_1, a_2, c_1, c_2) there exist control parameter values k and τ such that x^* is a stable equilibrium point of the controlled system. Furthermore, given an admissible tuple (a_1, a_2, c_1, c_2), conditions are established to identify such values of k and τ (i.e. the stabilizing control parameter values domain), although these conditions turn out to be overly restrictive.

In [3], the set of stabilizing values is analytically calculated for a system with an unstable focus perturbed by two scalar Pyragas-type controls.

We have reformulated the methodology of [3] for the two-dimensional HMN problem. Using this framework, we can obtain a complete description of the set of stabilizing values of the control parameters through explicit analytical expressions for the boundary curves of the region in the (k, τ) plane.

Trabajo en conjunto con González, Graciela Adriana (Universidad de Buenos Aires, Facultad de Ingeniería, Departamento de Matemática, Grupo de Matemática en Dinámicas No Lineales, Argentina) y Pastor, Verónica Estela (Universidad de Buenos Aires, Facultad de Ingeniería, Departamento de Matemática, Grupo de Matemática en Dinámicas No Lineales, Argentina).

Referencias

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