Homogenization theory for linear gravity waves over a non-uniform bottom with localized initial source

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We consider the linear potential model for the propagation of surface water waves over long distances in a basin with a variable bottom. In particular, we focus on the case where the bottom contains rapidly oscillating components that significantly influence the profile of the propagating wave.

We address the problem of describing surface waves generated by smooth, localized initial data. To this end, we formulate an auxiliary homogenization problem: to construct an equation with smooth coefficients whose solution approximates that of the original system.

We reduce the original system of equations to an evolution equation posed on the free surface by means of the Dirichlet-to-Neumann operator. The resulting equation is governed by a pseudo-differential operator with rapidly oscillating coefficients reflecting the fine-scale structure of the bottom.

We then apply a homogenization procedure based on operator separation of variables (adiabatic approximation) in order to derive an effective equation with smooth coefficients. Finally, we apply semi-classical asymptotic analysis to obtain explicit asymptotic formulas for the solution of the Cauchy problem for the homogenized equation, describing the propagation of waves over a rapidly oscillating bottom.

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Referencias

[1] S. Yu. Dobrokhotov, V. V. Grushin, S. A. Sergeev, B. Tirozzi: Asymptotic theory of linear water waves in a domain with nonuniform bottom with rapidly oscillating sections. // Russian Journal of Mathematical Physics 10/2016; 23(4):455-474., DOI:10.1134/S1061920816040038