A two-direction Frank-Wolfe method for multiobjective optimization on polytopes

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Constrained multiobjective optimization problems consist in minimizing simultaneously multiple objective functions over a feasible set $\mathcal{M} \subset \mathbb{R}^n$. Since in general these objectives are conflicting, they do not share the same minimizer and another notion of optimality is considered: a point in $\mathcal{M}$ is called Pareto optimal if there is no other point in $\mathcal{M}$ which improves at least one objective without increasing another.

In this work we consider $\mathcal{M}$ as a polytope defined by the convex hull of a finite set of points (atoms) and assume the objective functions are convex and continuously differentiable. A well known method for this setting is the classical Multiobjective Frank-Wolfe (M-FW) method which achieves a sublinear convergence rate [1, 3]. One of its variants, called Away-Step Frank-Wolfe (M-ASFW) introduces an *away* search direction and achieves linear convergence [2] in case the objectives are strongly convex.

We propose a new **Two-Direction Multiobjective Frank-Wolfe (TD-M-FW)** method which at each iteration chooses between two possible search directions: the classical Frank-Wolfe direction and an alternative one, defined as the difference between two points; one in $\mathcal{M}$ and another in the convex hull of the current active set of atoms. The choice is for the direction that yields the best improvement according to a suitable criterion. We show that this method generalizes both M-FW and M-ASFW.

Under strong convexity of the objectives and an algorithmic assumption on the step sizes, we show that the sequence generated by the resulting algorithm converges to a Pareto optimal point, and that a merit function introduced in [**?**] converges Q-linearly to zero. This analysis relies on the restricted width, a geometric property of polytopes.

Finally, we propose a specific alternative direction inspired by the pairwise Frank–Wolfe direction from scalar optimization. This strategy allows us to remove the assumption on the step sizes while preserving convergence: the sequence still converges to a Pareto optimal point and the merit function converges R-linearly to zero.

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Referencias

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