Weak Minimizing Property and Compact Perturbation

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For a bounded linear operator $T : X \to Y$, put $m(T) = \text{ínf}\{\|Tx\| : x \in S_X\}$. A sequence (x_n) in S_X is said to be minimizing for T if $\|Tx_n\| \to m(T)$. A pair (X, Y) has the weak minimizing property (WmP), introduced by Chakraborty in [2], if every operator $T : X \to Y$ admitting a non-weakly null minimizing sequence attains its minimum modulus $m(T)$. More recently, Han in [5] introduced the Compact Perturbation Property for the minimum modulus (CPPm), which requires that, for every operator $T : X \to Y$ that does not attain its minimum modulus, $\sup_{K\in\mathcal{K}(X,Y)} m(T + K) = m(T)$. In [5], it is shown that (ℓ_1, ℓ_1) fails both properties, while (c_0, c_0) fails the WmP. However, whether (c_0, c_0) has the CPPm was left open in Problem 3.6 of [5]. In this work, we answer this question negatively by proving that (c_0, c_0) does not have the CPPm. Our proof is constructive: we exhibit a non-minimum-attaining operator whose minimum modulus is strictly increased by a rank-one compact perturbation. Moreover, we show that this phenomenon is not specific to c_0: if $X = \mathbb{K} \oplus_\infty Y$, where Y is non-reflexive, then the pair (X, X) fails the CPPm.

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