Frames in shift invariant spaces of weighted mixed Lebesgue spaces $L^{p,q}_\mu$

Carolina Mosquera
Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires e IMAS-CONICET, Argentina
caroalemosquera@gmail.com

Frames were introduced by Duffin and Schaeffer for nonharmonic Fourier series in [2]. The interest in frame theory is due to its use in wavelets theory, time frequency analysis, and sampling theory. In these contexts, frames have some additional structure, such as generating a shift invariant space. The notion of Riesz basis has been generalized from $L^2(\mathbb{R}^d)$ to $L^p(\mathbb{R}^d)$ and studied in the context of shift invariant spaces by Jia et al. in [3]. Although the concept of frame in Banach spaces was studied in the 90's, its analysis in $L^p(\mathbb{R}^d)$, for p not equal to 2, was studied by Aldroubi et al. in [1]. In this work, the authors, given a set of generators of a shift invariant space in $L^p(\mathbb{R}^d)$, mainly give necessary and sufficient conditions for the set of generators to constitute a $p-$frame, and to generate a closed shift invariant subspace of $L^p(\mathbb{R}^d)$.

Weighted shift invariant spaces $V^p_\mu(\Phi)$, $p \in [1, +\infty]$, where μ is a weight, were introduced for non uniform sampling as a direct generalization of the space $V^p(\Phi)$ ([1],[5]). In [4], the authors show that the main result of [1] also holds in the case of weighted shift invariant spaces.

Some signals are time-varying in practice, meaning that the signals live in time-space domains at the same time. The mixed Lebesgue space is a suitable tool for modeling and measuring time-space signals, due to the separate integrability for different variables. In fact, function spaces with mixed norms are of great practical significance and have developed rapidly, finding applications across many fields of mathematics. It is known that $L^{p,q}(\mathbb{R}\times\mathbb{R}^d)$ inherits many excellent properties of the traditional $L^p(\mathbb{R}^d)$ space. However, the order of integration in the definition of $L^{p,q}(\mathbb{R}\times\mathbb{R}^d)$ is not commutative. Therefore, research on some issues in mixed-norm Lebesgue spaces also brings various challenges.

In this talk we consider $(p, q)-$frames in shift-invariant spaces

$$V^{p,q}_\mu(\Phi) = \left\{ f \in L^{p,q}_\mu : f = \sum_{i=1}^{r} \sum_{j_1\in\mathbb{Z}} \sum_{j_1\in\mathbb{Z}^d} d_i(j_1, j_2)\phi_i(. - j_1, . - j_2), (d_i(j_1, j_2)) \in \ell^{p,q}_\mu(\mathbb{Z}\times\mathbb{Z}^d) \right\},$$

of weighted mixed Lebesgue spaces $L^{p,q}_\mu(\mathbb{R}\times\mathbb{R}^d)$, where $\Phi = \{\phi_1, \cdots, \phi_r\}$. We prove the equivalence of the frame property and the closedness for the weighted shift-invariant subspace $V^{p,q}_\mu(\Phi)$.

Trabajo en conjunto con Rocío Balderrama (Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires e IMAS-CONICET, Argentina).

Referencias

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