Using quasi-Darboux transformations to construct exceptional matrix polynomials

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Exceptional polynomials are a generalization of the classical Hermite, Jacobi, and Laguerre polynomials. They are sequences $\{p_n(x)\}$ of polynomials that are orthogonal with respect to a positive weight $w(x)$ and are also eigenfunctions of a second-order differential operator

$$p_n(x) \cdot D = p_n''(x)\, f_2(x) + p_n'(x)\, f_1(x) + p_n(x)\, f_0(x) = \lambda_n\, p_n(x),$$

while allowing finitely many degrees (the exceptional degrees) to be missing from the sequence. In this talk, based on [1], we consider a matrix-valued extension of these exceptional families and introduce a new method, which we call the quasi-Darboux transformation, for constructing exceptional matrix-valued orthogonal polynomials. In contrast with the scalar setting, the noncommutativity of matrix coefficients introduces additional difficulties in the construction process. We will illustrate the quasi-Darboux approach through some explicit examples.

Trabajo en conjunto con Ignacio Zurrián (Universidad de Sevilla, España) y Antonio Durán (Universidad de Sevilla, España).

Referencias

[1] Ignacio Bono Parisi, Antonio Durán y Ignacio Zurrián, "Using quasi-Darboux transformations to construct exceptional matrix polynomials", Preprint, arXiv:2510.13487 (2025)