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Musielak-Orlicz spaces L^Φ generalize Orlicz spaces L^Ψ and variable exponent Lebesgue spaces $L^{p(\cdot)}$. By allowing the modular $\Phi(x, t)$ to depend on the spatial variable x , Musielak-Orlicz spaces provide a flexible framework for modeling nonstandard growth and heterogeneous regularity. Consequently, they have become an important tool in nonlinear analysis, the study of partial differential equations with nonstandard growth, and the calculus of variations.

Expanding on previous work, we develop a general theory of best modular approximation in Musielak-Orlicz spaces under minimal regularity assumptions on the modular. Our approach neither requires differentiability of Φ with respect to its second variable nor excludes piecewise affine behavior near the origin. This level of generality allows us to cover important classes of Musielak-Orlicz spaces that fall outside the scope of the existing theory.

For this presentation, we introduce a novel classification of Orlicz functions according to their behavior near the origin: initially linear, initially strict and vanishing staircase. This allows us to conveniently partition the compact domain of Musielak-Orlicz functions. Furthermore, this leads to uniqueness results for the best approximation by linear subspaces in R and R^d .

Trabajo en conjunto con Sergio Favier (Universidad Nacional de San Luis, Argentina) y Fabián Levis (Universidad Nacional de Río Cuarto, Argentina).

Referencias

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