Irreducibility of eigenspaces of the Laplacian for generic invariant metrics

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Let M be a closed (compact and without boundary) differentiable manifold of dimension at least 2. Karem Uhlenbeck [1] proved that, for a **generic** Riemannian metric g on M, the spectrum of the Laplace–Beltrami operator Δ_g is simple; that is, all of its eigenspaces are one-dimensional.

Now suppose that a compact Lie group G acts smoothly on M. If g is a G-invariant Riemannian metric, then Δ_g commutes with the action of G, and hence each eigenspace of Δ_g naturally carries the structure of a G-module. As a consequence, eigenspaces are no longer one-dimensional in general.

In this setting, Shing-Tung Yau [2, Problem 42] proposed to study under what conditions, for a generic G-invariant metric on M, every eigenspace of Δ_g is irreducible as a G-module.

In this talk, we will present an overview of the progress made on this problem and discuss some recent results for homogeneous spaces, obtained in joint work with Fiorela Rossi Bertone. In a complementary talk, she will explain the main ideas behind the proofs, which rely almost entirely on the representation theory of compact Lie groups.

Trabajo en conjunto con Fiorela Rossi Bertone (Universidad Nacional del Sur, Argentina).

Referencias

[1] K. Uhlenbeck. ”Generic properties of eigenfunctions”. Amer. J. Math. 98:4 (1976), 1059-1078.

[2] S.-T. Yau. .ºpen problems in geometry”. In Differential geometry: partial differential equations on manifolds (Los Angeles, CA, 1990), 1-28, Proc. Sympos. Pure Math. 54, Amer. Math. Soc., Providence, RI, 1993.