On the Existence of Transversals and Near Transversals in Groups, Loops and Quasigroups

Adriana Juzga León
Universidade do Estado do Rio de Janeiro, Brasil
ajuzgal@gmail.com

A Latin square of order n is an $n \times n$ array in which each row and each column is a permutation of a set of n symbols. The multiplication tables of several algebraic structures, such as groups, quasigroups, and loops, are natural examples of Latin squares. A partial transversal of a Latin square is a set of entries lying in distinct rows and columns and containing distinct symbols. If the Latin square has order n, a partial transversal is called a near transversal if it has size $n-1$, and a transversal if it has size n.

In this talk, we investigate how the existence of these subsets in Latin squares is related to algebraic properties of the underlying structures, focusing on classical results for Latin squares arising from finite groups. We will also discuss possible extensions of these results to loops and, more generally, to certain special classes of loops and quasigroups, with the aim of understanding to what extent combinatorial properties of Latin squares reflect structural properties of these algebraic systems.

Trabajo en conjunto con Dylene Agda Souza de Barros (Universidade Federal de Uberlândia, Brasil) y Rosemary Miguel Pires (Universidade Federal Fluminense, Brasil).

Referencias

[1] K. Balasubramanian, On transversals in Latin squares, Linear Algebra Appl. 131 (1990), 125–129

[2] J. Dénes and A. D. Keedwell, Latin squares: New developments in the theory and applications, Ann. Discrete Math., 46, North-Holland, Amsterdam, 1991

[3] L. Goddyn and K. Halasz, All group-based Latin squares possess near transversals, J. Combin. Des., 28 (2020), 358–365.

[4] M. Hall and L. J. Paige, Complete mappings of finite groups, Pacific Journal of Mathematics 5 (1955), 541–549.