Half-automorphism group of some loops

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A *loop* is a set L with a binary operation $\cdot$ and a neutral element $1 \in L$ such that for every a, $b \in L$ the equations $ax = b$ and $ya = b$ have unique solutions x, $y \in L$, respectively.

Let $(L, *)$ and $(L', \cdot)$ be loops. A bijection $f : L \to L'$ is a *half-isomorphism* if $f(x * y) \in \{f(x) \cdot f(y), f(y) \cdot f(x)\}$, for every x, $y \in L$. A *half-automorphism* is defined as expected. We say that a half-isomorphism (half-automorphism) is *nontrivial* if it is neither an isomorphism (automorphism) nor an anti-isomorphism (anti-automorphism).

In 1957, W.R. Scott proved every half-isomorphism of a cancellation semi-group into a cancellation semi-group is either an isomorphism or an anti-isomorphism. In particular, there is no nontrivial half-homomorphism between two groups. He also gave an example of a loop of order 8 that has a nontrivial half-automorphism, so Scott's result can not be generalized to all loops.

In this talk we will present some loops for which Scott's result still holds, and we will discuss about the group of half-automorphisms of code loops and a class of Bol loops that contain nontrivial half-automorphisms.