A convergent numerical scheme for the porous medium equation with fractional pressure

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We introduce and analyze a numerical approximation of the porous medium equation with fractional potential pressure introduced by Caffarelli and Vázquez:

$$\partial_t u = \nabla \cdot (u^{m-1} \nabla (-\Delta)^{-\sigma} u) \qquad \text{for} \qquad m \geq 2 \quad \text{and} \quad \sigma \in (0,1).$$

Our scheme is for one space dimension and positive solutions u. It consists of solving numerically the equation satisfied by $v(x,t) = \int_{-\infty}^{x} u(y,t)dy$, the quasilinear nondivergence form equation

$$\partial_t v = -|\partial_x v|^{m-1} (-\Delta)^s v \qquad \text{where} \qquad s = 1 - \sigma,$$

and then computing $u = v_x$ by numerical differentiation. Using upwinding ideas in a novel way, we construct a new and simple, monotone and L^∞-stable, approximation for the v-equation. The full scheme then becomes a conservative up-wind finite volume approximation for the u-equation. We show local uniform convergence to the unique discontinuous viscosity solution for the v-problem, and using ideas from probability theory, we prove that the approximation of u converges up to normalization in $C(0,T;P(\mathbb{R}))$ where $P(\mathbb{R})$ is the space of probability measures under the Rubinstein-Kantorovich (bounded Lipschitz) metric. The analysis includes also fundamental solutions where the initial data for u is a Dirac mass.

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