On the robustness and fully discrete entropy stability of three-point well-balanced finite-volume schemes

Manuel Jesús Castro Díaz
Universidad de Málaga. Dpto. Análisis Matemático, Estadística e Investigación Operativa y Matemática Aplicada, España
mjcastro@uma.es

This presentation introduces a simple and robust approach to enforce discrete entropy stability in first-order well-balanced finite volume schemes for systems of balance laws, including those with non-conservative terms. Building on Tadmor's artificial viscosity method and its recent extensions, the authors present an entropy-preserving modification that can be applied to a broad class of three-point schemes. The key contribution is the design of a viscosity coefficient that preserves the well-balanced property while ensuring entropy dissipation at the discrete level. A rigorous theoretical framework is established to guarantee robustness, well-balancing, and entropy stability under an appropriate CFL condition. The effectiveness of the method is demonstrated through numerical experiments on shallow water systems and two-layer flows, confirming both accuracy and stability

Trabajo en conjunto con Christophe Berthon (Université de Nantes, Laboratoire de Mathématiques Jean Leray. Francia), Ludovic Martaud (Université Rennes, Inria Rennes and IRMAR UMR CNRS 6625, F-35042 Rennes, Francia) y Tomás Morales de Luna (Universidad de Málaga, Dpto Análisis Matemático, Estadística e Investigación Operativa y Matemática Aplicada. España).

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