THE MODULI SPACE OF SINGULAR PRINCIPAL BUNDLES OVER THE MODULI SPACE OF STABLE CURVES

Alexander Schmitt

Freie Universität Berlin, Alemania

alexander.schmitt@fu-berlin.de

In the study of moduli spaces of vector or principal bundles over smooth projective curves and their properties, one may use degenerations to singular curves. Motivated by this, Bhosle [3] and the speaker [9] constructed moduli spaces of singular principal bundles over irreducible curves with only nodes as singularities. The analog for reducible curves has been considered in the thesis of Ángel Muñoz Castañeda [5].

For a given semisimple structure group G and genus $g \geq 2$, there is a universal moduli space $\mathcal{M}_{g,G}$ of semistable principal G-bundles over the moduli space $\mathcal{M}_g$ of smooth curves of genus g. Using the aforementioned results, Muñoz Castañeda and the speaker [6, 7] constructed a moduli space of singular principal G-bundles on stable curves which compactifies $\mathcal{M}_{g,G}$ relative to the moduli space $\overline{\mathcal{M}}_g$ of stable curves, generalizing Pandharipande's [8] construction for the structure group GL_n. Compactifications of $\mathcal{M}_{g,G}$ which are flat over $\mathcal{M}_g$, but do not have a modular interpretation were obtained by Manon [4] and Belkale/Gibney [2] for the structure group $G = \mathrm{SL}_n$, and by Wilson [10] for simple and simply connected Lie groups of type A or C, using vector bundles of conformal blocks. Anderson, Esole, Fredrickson, and Schaposnik [1] have raised similar questions for Higgs bundles in view of possible applications to string theory.

In this talk, I will present the joint work with Muñoz Castañeda and briefly discuss Wilson's work on the relation of our moduli space and conformal blocks.

Trabajo en conjunto con Ángel L. Muñoz Castañeda (Universidad de León, España).

Referencias

[1] L.B. Anderson, M. Esole, L. Fredrickson, L. Schaposnik, Singular geometry and Higgs bundles in string theory, SIGMA Symmetry Integrability Geom. Methods Appl. 14 (2018), paper no. 037, 27 pp.

[2] P. Belkale, A. Gibney, On finite generation of the section ring of the determinant of cohomology line bundle, Trans. Amer. Math. Soc. 371 (2019), no. 10, 7199-242.

[3] U.N. Bhosle, Tensor fields and singular principal bundles, Int. Math. Res. Not. 2004, no. 57, 3057-77.

[4] Ch. Manon, Coordinate rings for the moduli stack of $\mathrm{SL}_2(C) quasi-parabolic principal bundles on a curve and toric fibrations$ 83.

[5] A.L. Muñoz Castañeda, On the moduli spaces of singular principal bundles on stable curves, Adv. Geom. 20 (2020), no. 4, 573-84.

[6] A.L. Muñoz Castañeda, A compactification of the universal moduli space of principal G-bundles, Mediterr. J. Math. 19 (2022), no. 2, paper no. 54, 23 pp.

[7] A.L. Muñoz Castañeda, A.H.W. Schmitt, Singular principal bundles on reducible nodal curves, Trans. Amer. Math. Soc. 374 (2021), no. 12, 8639-660.

[8] R. Pandharipande, A compactification over $\overline{M}_g of the universal moduli space of slope-semistable vector bundles, J.A$ 71.

[9] A.H.W. Schmitt, Moduli spaces for semistable honest singular principal bundles on a nodal curve which are compatible with degeneration. A remark on U.N. Bhosle's paper:

[10] A. Wilson, Compactifications of moduli of G-bundles and conformal blocks, arXiv:2104.07549, 25 pp.